Entropic model for real-time dose calculation: an application in brachytherapy

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Déclaration Publique d’Intérêts

Durant les cinq dernières années:
Je ne déclare aucun lien d’intérêt
Outline

- Introduction
- The M1 model
- Applications to brachytherapy
- Conclusions and perspectives
Introduction
Overview

- Significant developments have increased the use of **3D image guided procedures** with the utilization of CT, MRI, US, and PET;

- In this direction it could be interesting to have an algorithm that can calculate **precisely and instantaneously** the dose distribution during the treatment;

- **Monte-Carlo method** is a possible solution to attend the requested accuracy but is very time consuming;

- Other algorithms as Pencil Beam or Collapsed Cone Convolution are really **fast but can be inaccurate** in case of complex geometry;

A new type C algorithm could be a good compromise between Monte-Carlo and Collapsed Cone Convolution algorithms.
The deterministic method is based on the 3 dimensional linear Boltzmann transport equation (LBTE), calculating the particle distribution function:

\[
\Omega \cdot \nabla \cdot \psi^i + \sigma_i^j \psi^j = \sum_{p=[i,j]} \int \int_{S^2} d\Omega \int d\varepsilon \sigma^{p,i} (\varepsilon \rightarrow \varepsilon', \Omega \rightarrow \Omega') \psi^i (r, \varepsilon, \Omega)
\]

To be numerically solved, the LBTE has to be discretized:

- **Space (voxel)**
- **Energy**
- **Angle**

The LBT equation has 6 variables. To be numerically solved and to reduce the computational time, approximations are needed.
M\textsubscript{1} deterministic model

The moments of the equation

The deterministic method is based on the 3 dimensional linear Boltzmann transport equation (LBTE), calculating the particle distribution function:

\[ \Omega \cdot \nabla \cdot \psi^i + \sigma^i \psi^i = \sum_{p=[i,j]} \int d\varepsilon \int_{S^2} d\Omega \sigma^{p,i} (\varepsilon \to \varepsilon', \Omega \to \Omega') \psi^i (r, \varepsilon, \Omega) \]

With the aim of reducing the number of variables, with the M1 model we study the angular moments of the equation instead of the flux itself.

\[
\begin{align*}
\psi^i_0 (r, \varepsilon) &= \int_{S^2} \psi^i (r, \varepsilon, \Omega) d\Omega, \\
\psi^i_1 (r, \varepsilon) &= \int_{S^2} \Omega \cdot \psi^i (r, \varepsilon, \Omega) d\Omega, \\
\psi^i_2 (r, \varepsilon) &= \int_{S^2} (\Omega \otimes \Omega) \psi^i (r, \varepsilon, \Omega) d\Omega
\end{align*}
\]

Studying the angular moments of the equation we lose the angular dependency.
The kinetic equations for the first two moments are:

\[
\sigma_i^j(\epsilon')\psi^i_0(r, \epsilon') + \nabla \cdot \psi^i_1(r, \epsilon') = \sum_{p=[i,j]} \int \sigma_0^{p,i}(\epsilon \rightarrow \epsilon') \psi^i_0(r, \epsilon) d\epsilon,
\]

\[
\sigma_i^j(\epsilon')\psi^i_1(r, \epsilon') + \nabla \cdot \psi^i_2(r, \epsilon') = \sum_{p=[i,j]} \int \sigma_1^{p,i}(\epsilon \rightarrow \epsilon') \psi^i_1(r, \epsilon) d\epsilon.
\]

The function that satisfies the Boltzmann’s H-theorem is an exponential:

\[
\psi^i_{ME} = a_0 e^{-\Omega a_1}
\]

where \(a_0 \geq 0\) is a scalar and \(a_1\) is a vector in \(\mathbb{R}^3\).

The distribution function is:

- subject to restriction of its degrees of freedom;
- generated by a non negative underlying distribution function.
M$_1$ deterministic model
The closure of the system

- Now we can write the second moment in term of the zeroth and first moments as:

\[
\psi^i_2(r, \epsilon) = \int_{S^2} (\Omega \otimes \Omega) \psi^i(r, \epsilon, \Omega) d\Omega
\]

\[
= \psi^i_0 \left( \frac{1 - \chi(\alpha)}{2} \mathbf{1} + \frac{3\chi(\alpha) - 1}{2} \frac{\psi^i_1}{|\psi^i_1|} \otimes \frac{\psi^i_1}{|\psi^i_1|} \right)
\]

- Where the Eddington factor is interpolated by:

\[
\chi = \frac{1}{3} \left( 1 + \alpha^2 + \alpha^4 \right) \quad \text{and} \quad |\alpha| = \frac{|\psi^i_1|}{\psi^i_0}
\]

- Finally we can calculate the dose distribution as:

\[
D(r) = \frac{T}{\rho(r)} \int_0^\infty S(r, \epsilon') \psi^{(0)}(r, \epsilon') d\epsilon'
\]

To calculate the dose only the zeroth moment is needed!
## M₁ deterministic model
### A comparison with the existing models

<table>
<thead>
<tr>
<th>Algorithm</th>
<th>Discretization in space</th>
<th>Discretization in energy</th>
<th>Discretization in directions</th>
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</thead>
<tbody>
<tr>
<td>Collapsed cone algorithm *</td>
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<td>X</td>
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<tr>
<td>Deterministic algorithm **</td>
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<td>X</td>
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<tr>
<td>Entropic algorithm ***</td>
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</tbody>
</table>

The entropic algorithm has a structural advantage on the other models already commercially available. It can bring together:

- accuracy of a deterministic algorithm;
- rapidity of a collapsed cone algorithm.

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** Gifford K et al. "Comparison of a finite element multigroup discrete ordinates code with Monte-Carlo for radiotherapy calculation" PMB (2006)
The M1 model has been already validated for external photon radiotherapy * even in presence of external magnetic field **. This will have an implication on the flux of charged particles, which has to be taken into account.

In brachytherapy the dose calculation is based on simplistic assumptions *

- The dose distribution is calculated in homogeneous liquid water phantom considering cylindrically symmetric sources;
- Inter-seed attenuation and inter-applicator shielding are not treated by current clinical calculations;
- The real chemical composition of the different tissues is not taken into account;
- The effects of patient dimension are not considered and may modify the dose distributions at the interfaces.

This approach is fast and practical in the clinical context but can lead to inaccuracies**.

If the chemical composition is taken into account:

- the pelvic bones can receive 70% more absorbed dose;
- the calcifications present in the prostate can reach 400% more dose;
- A lack of dose (up to 20%) appears in pixels where adipose is present.
Brachytherapy
Validation with Monte-Carlo simulations

Source: I-125
Meshing: 0,125 mm³
Time: 10 s /seed

Source: Ir-192
Meshing: 0,125 mm³
Time: 10 s /seed
Conclusions and on going work

Conclusions:

- M1 code can calculate the dose distributions with high precision in agreement with MC codes;
- M1 in term of ratio precision/calculation time should be better than any other deterministic code developed so far;
- M1 is structurally suitable both for external and for internal radiotherapy.

On going work:

- The code still has large margin to optimization: adaptive mesh refinement (AMR), higher order schemes, more efficient parallelization…
- Benchmark with others TPS: implementation of applicator geometry and realistic seeds geometry are needed
Thanks for your attention!